Stable Bases for Music in Time-Frequency Plane

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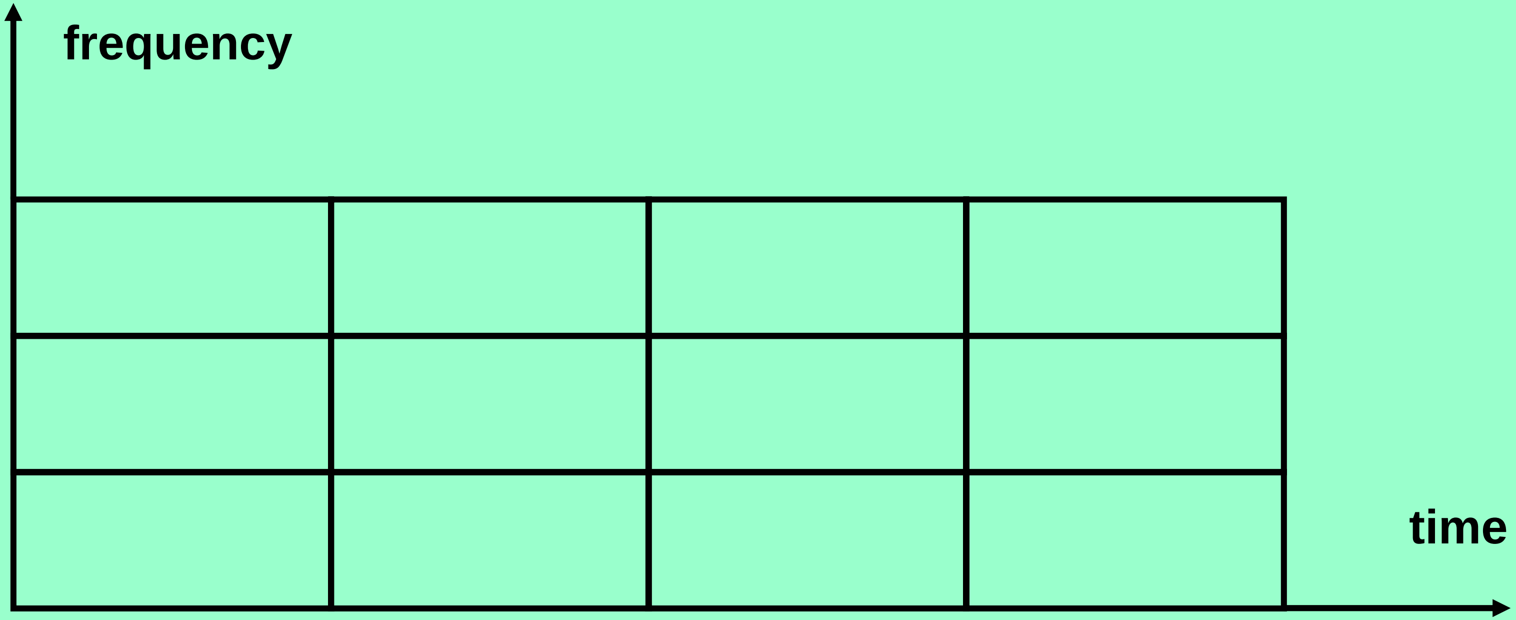
Harmonic Analysis and Applications, Merlo August 2006

Gabor Weyl-Heisenberg



$$\left\{ e^{i2\pi jbt} g(t-ka) \right\}$$

$ab = 1$ *unstable basis*

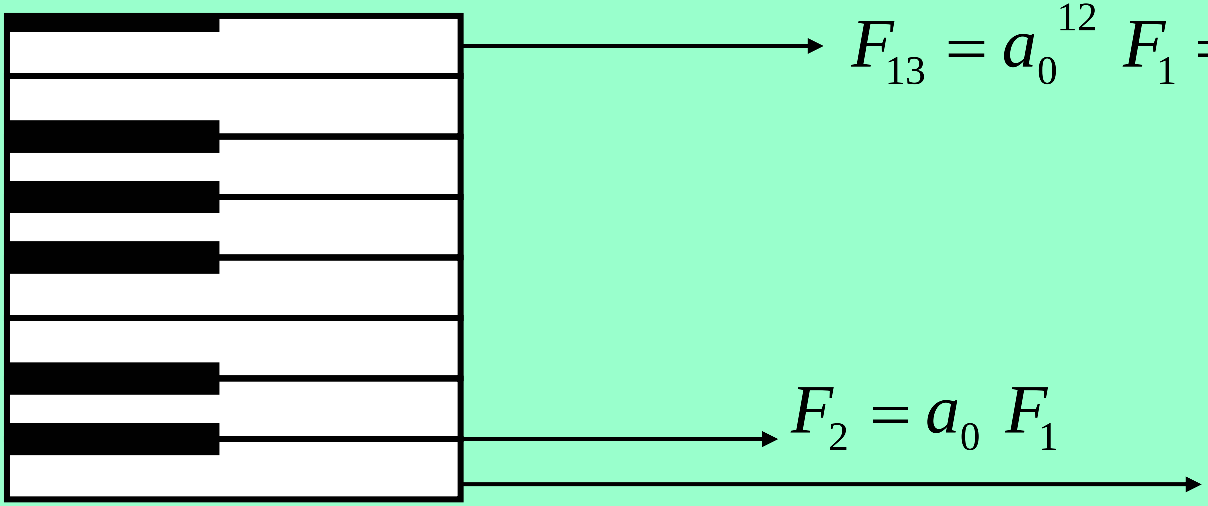
$ab < 1$ *stable frame*

Regular tiling

Optimal time-freq localization

HAA 2006

Music



$$F_{13} = a_0^{\ 12} F_1 = 2 F_1$$

$$F_2 = a_0 F_1$$

$$F_1$$

1 octave

geometric progression

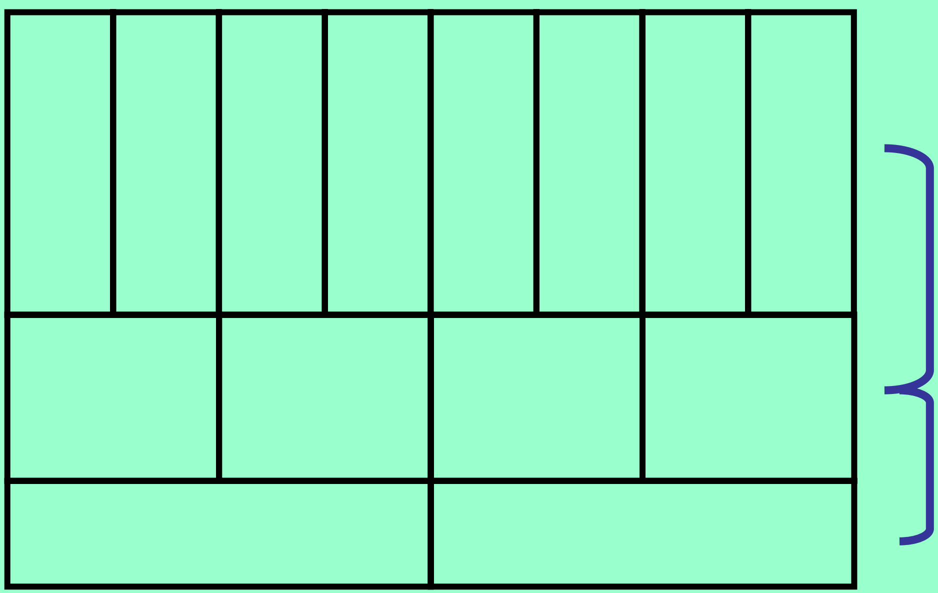
frequency ratio of 2 adjacent notes = constant

$a_0 = 2^{1/12}$ **irrational**

Discrete dyadic wavelets

Tiles: constant area

Longer tiles needed to identify lower frequencies



$$\left\{ \Psi(2^{j}\,t - k) \right\}$$

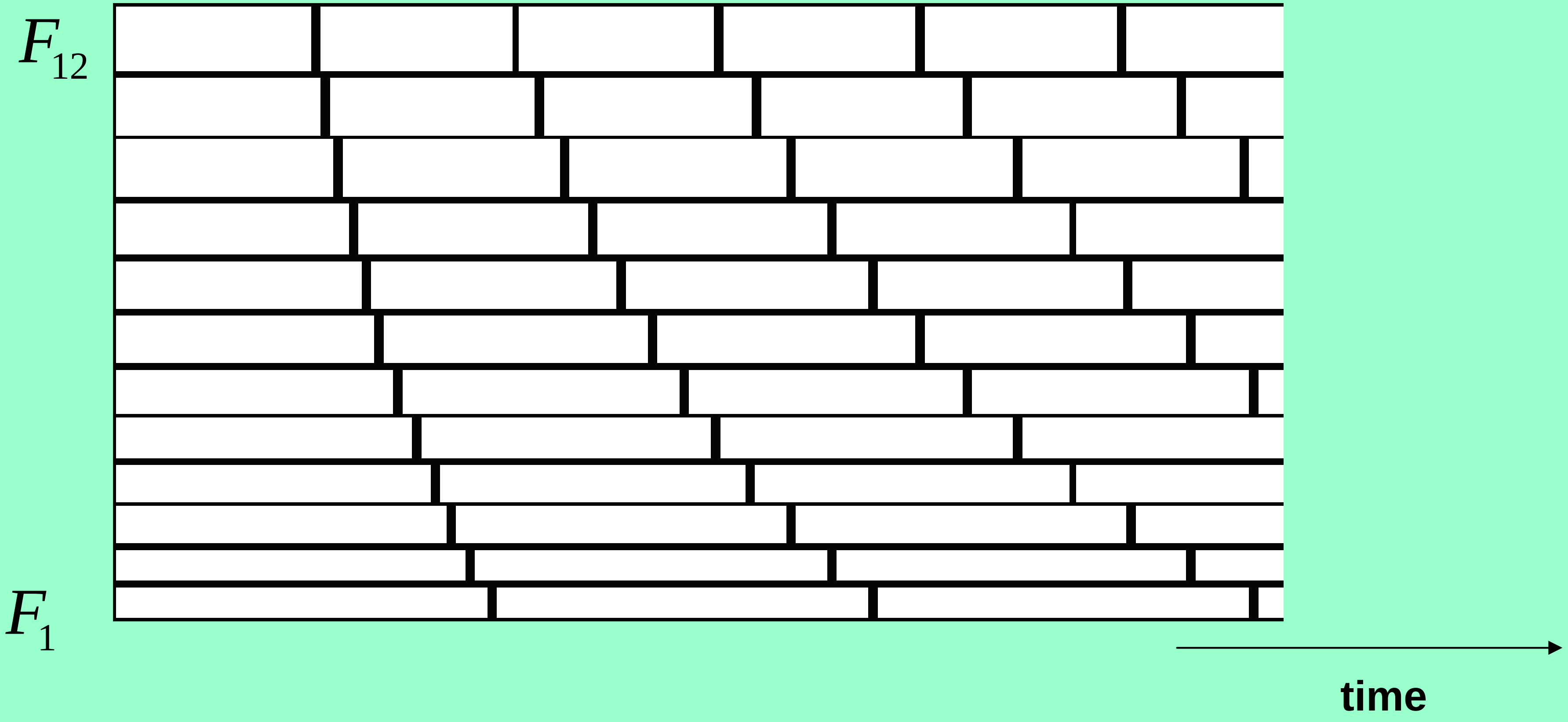
Stable basis

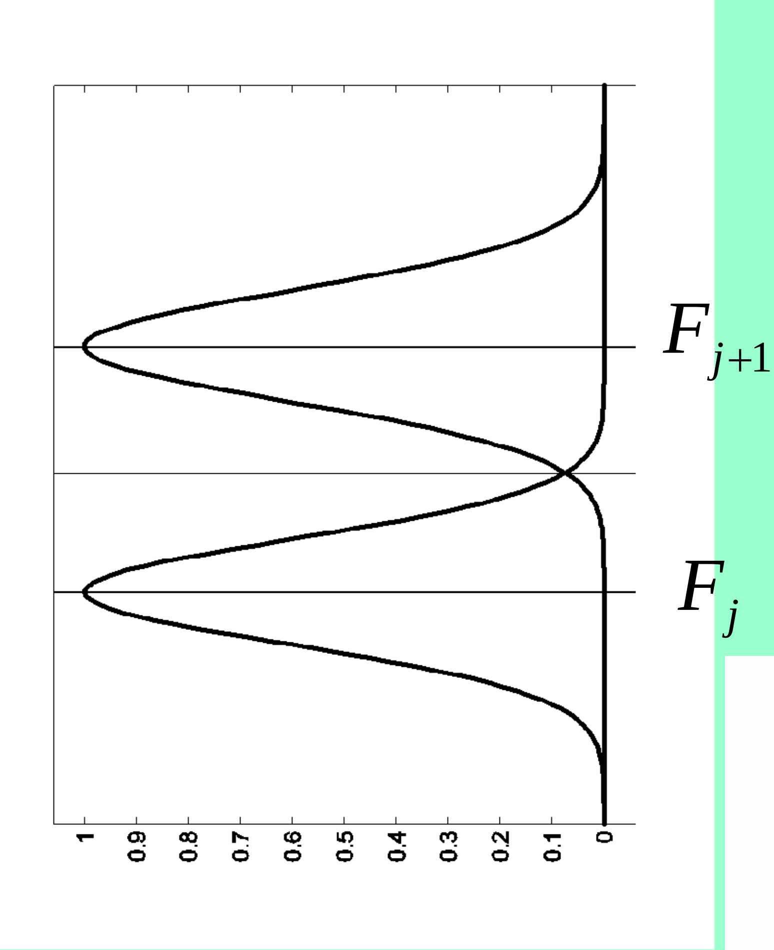
Excellent time localization

Not too good freq. localization

Special tiling for an octave

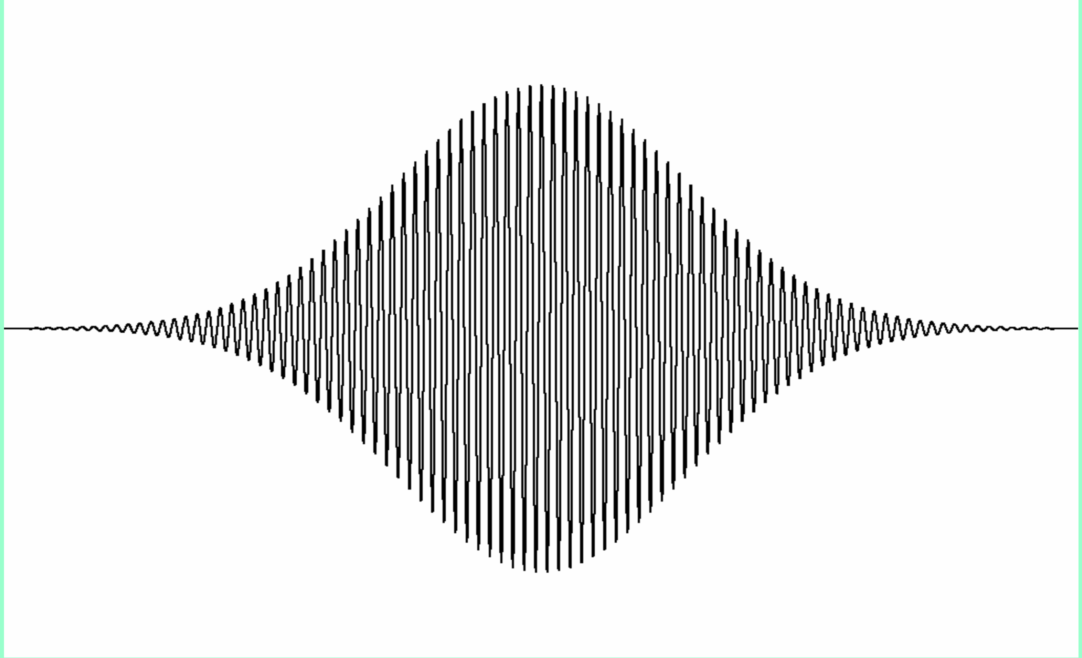
Tiles: constant area
Longer tiles needed to identify lower frequencies





Real Gabor Wavelet

on special tiling !



Goal:

reduce the inner products between basis functions

(I) Rational approximation of the tiling

↓

Orthogonality between odd shifts of the basis function

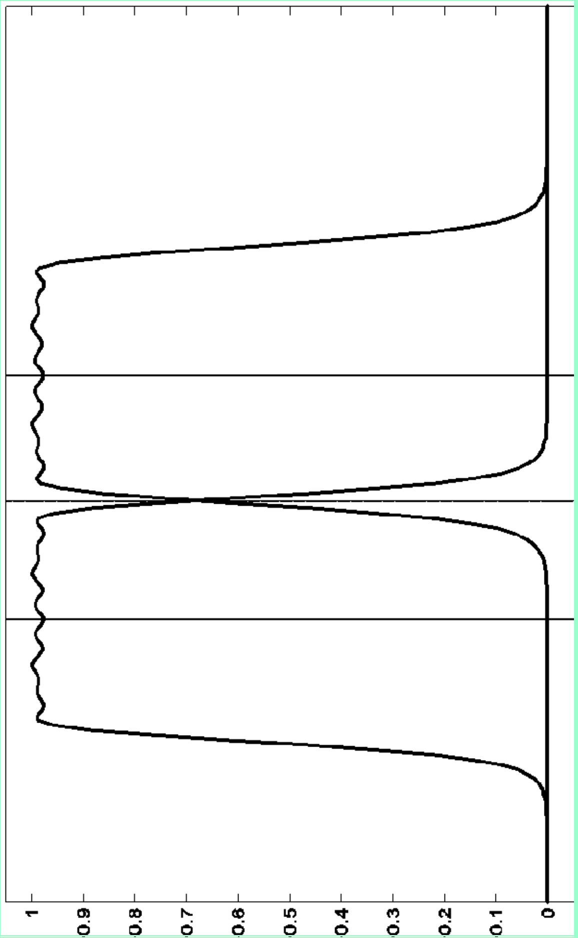
(II) Modification of the basis function

↓

All inner products below 0.01

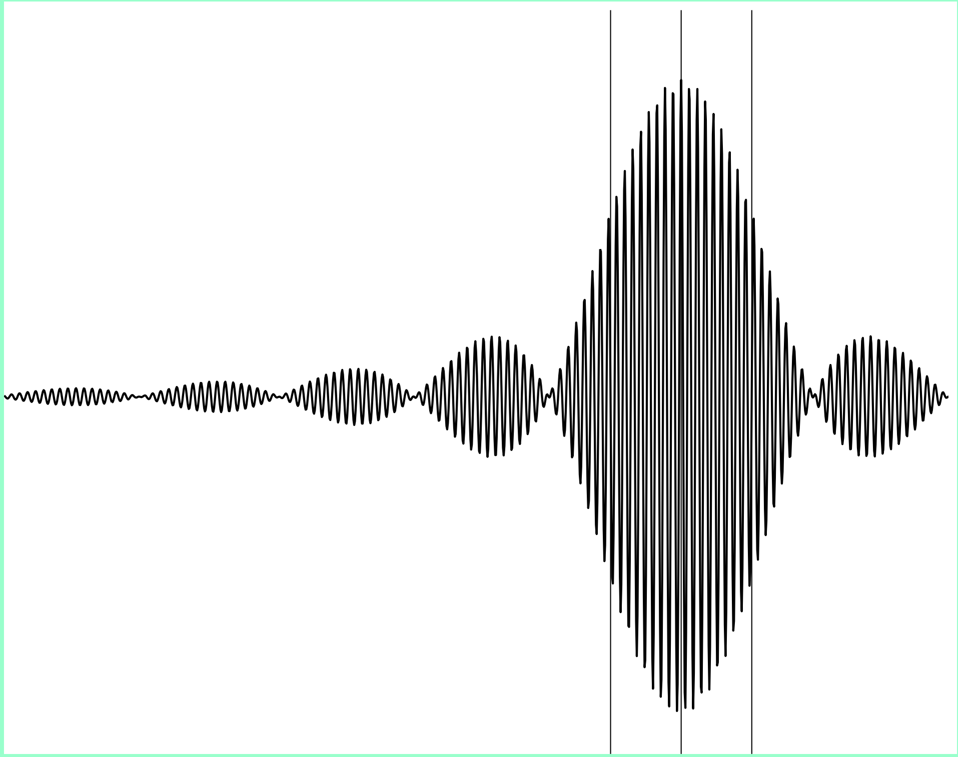
- - - - - - - - - - - - - - - → ***STABLE BASIS***

Modified Wavelet

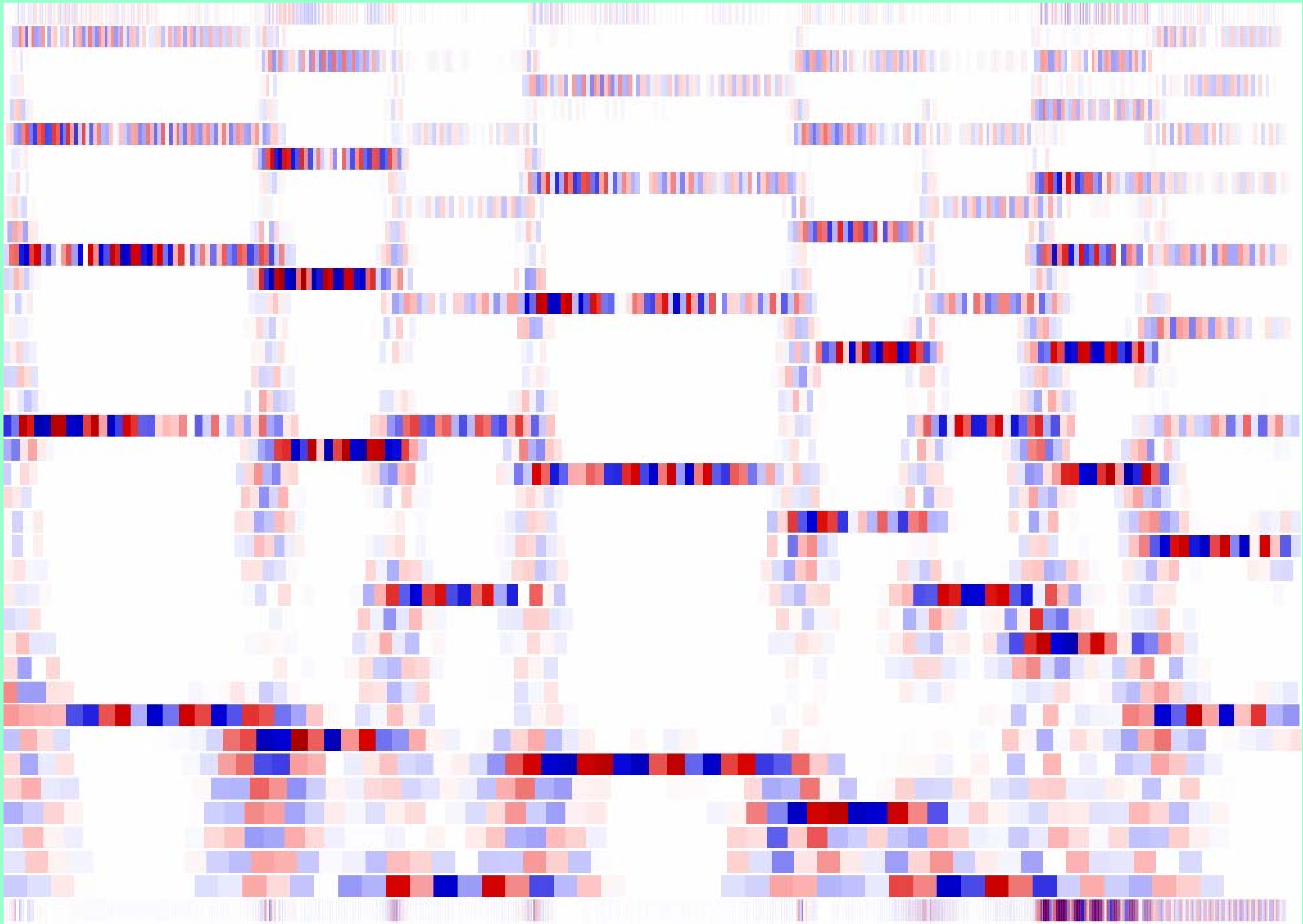


F_{j+1}

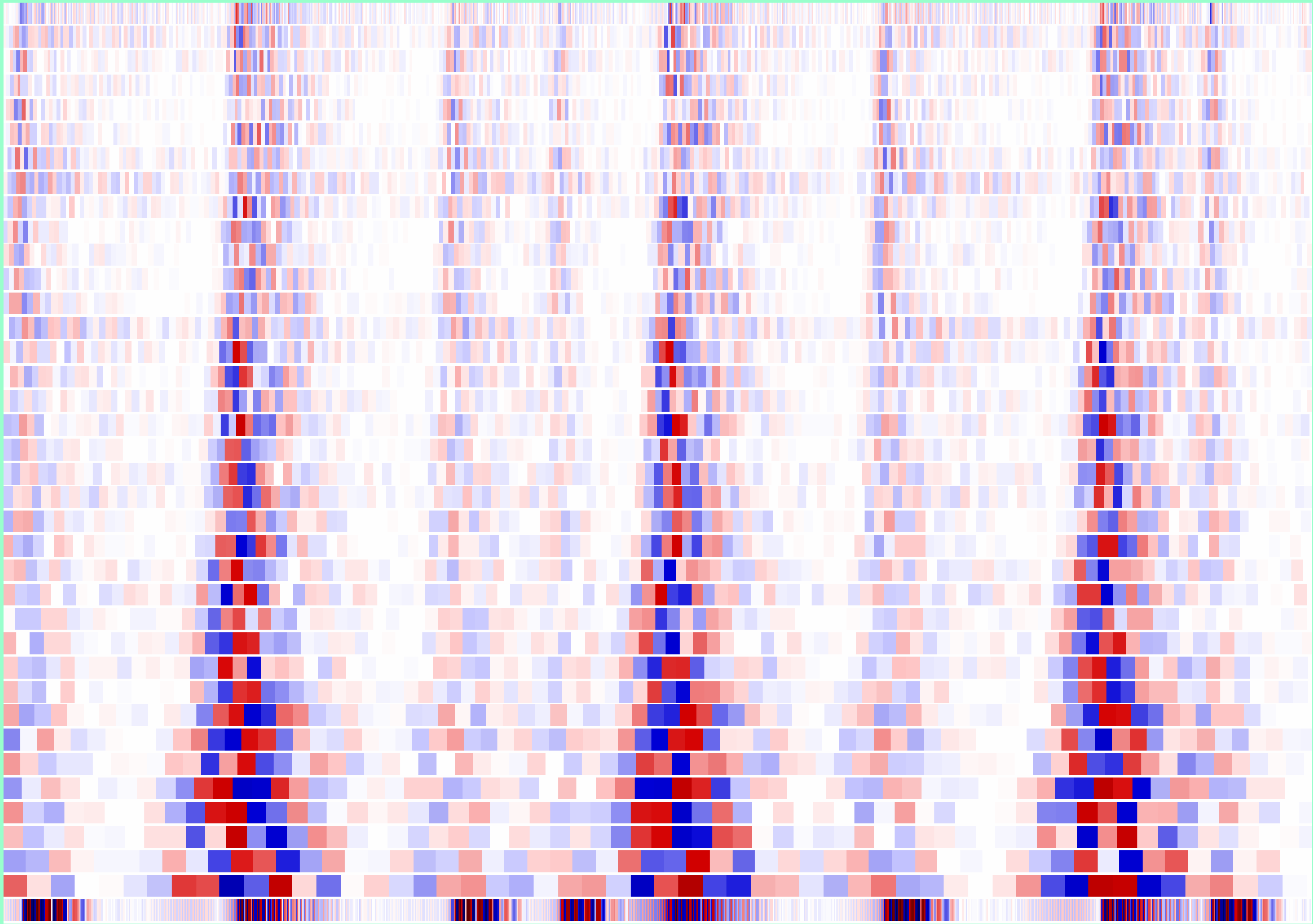
F_j



Tests a) Melody : Coefficients in our basis

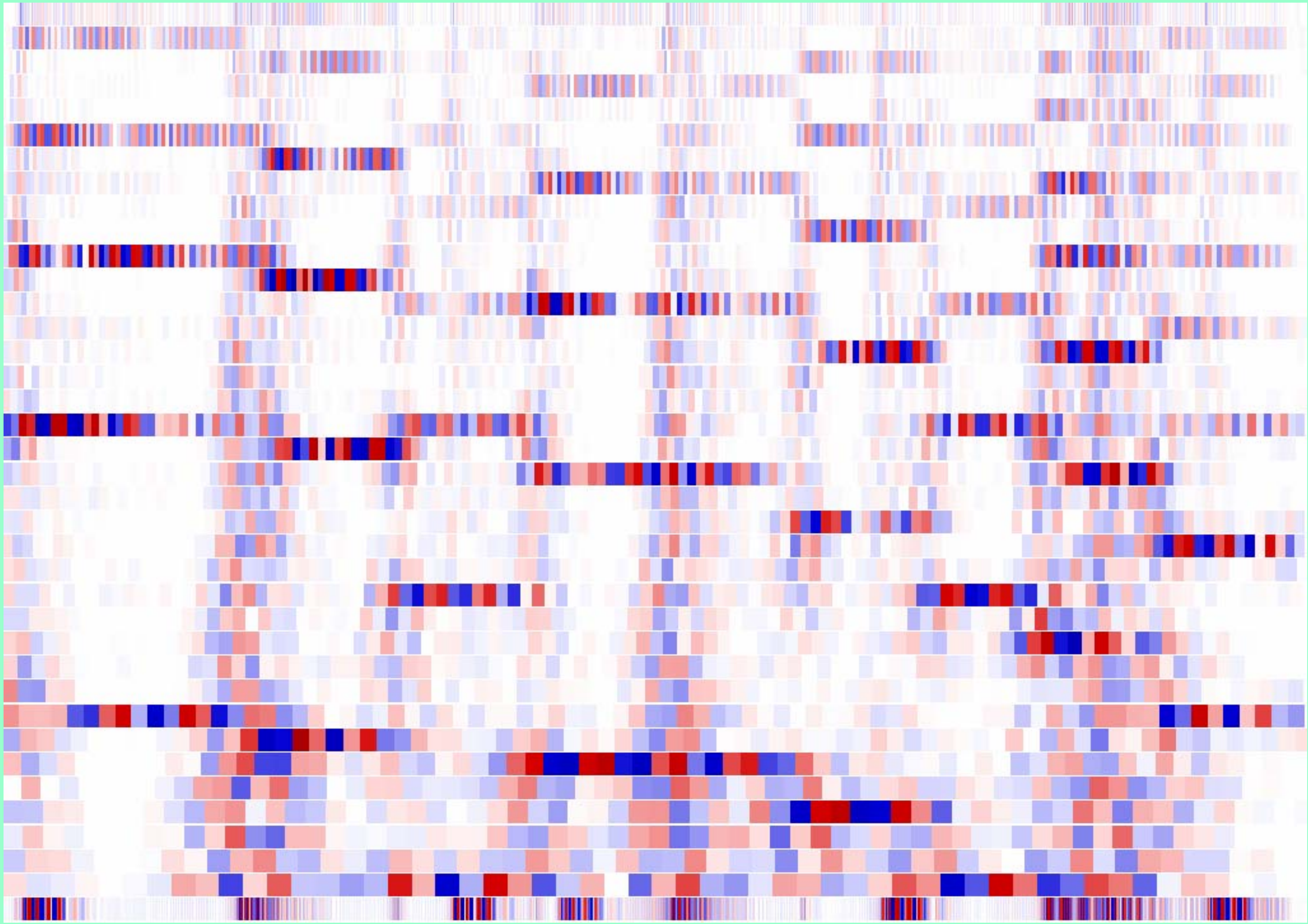


Tests b) Rythm : Coefficients in our basis

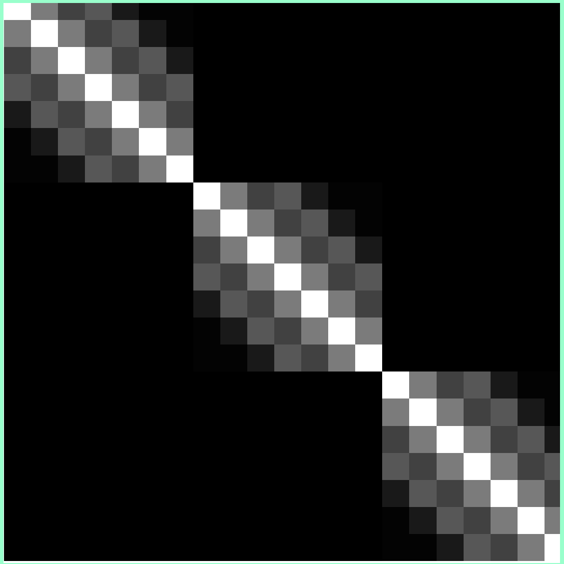


All frequencies at given times

Tests c) Mix : Coefficients in our basis

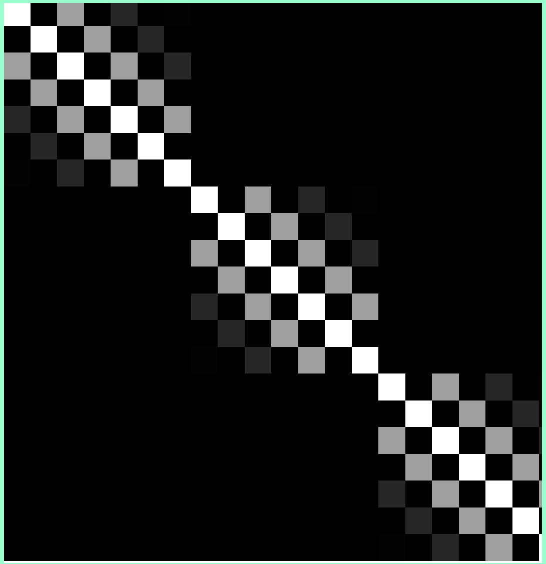


Autocorrelation matrix



Gabor wavelet

Irrational tiling



Gabor wavelet

Rational tiling

Modified wavelet

Rational tiling