"Orthonormal Bases and Tilings of the Time-Frequency Plane for Music Processing"

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Extended Abstract

Finding good bases for joint time-frequency representation of signals is a key problem in Signa Processing. The elements of these bases are called time-frequency atoms. These atoms have their energy concentrated in a small area of the time-frequency plane called a tile. The tiles have the same area and they form a partition of the plane. The time-frequency plane can be divided, for example, in horizontal bands, each covering a different frequency interval. Then, each band can be sliced independently in rectangles of appropriate size. None of the known techniques can produce bases tha provides an analysis band for each musical tone, with good separation from tones from neighbor bands. This means that some applications use overcomplete representations, and others use techniques that do not match music signals well. And many can't perfectly reconstruct the original signal.

Wavelets are the latest and most successful way to represent signals showing time and frequenc content at the same time. However; dyadic, rational, and M-ary wavelets can not be adjusted to the nature of musical scale tones. The tones of the chromatic tempered scale used in western music have an equal relative bandwidth (also called constant Q). M-ary wavelets are the first to be discarded, as they can't generate constant Q bands. Dyadic wavelets generate constant Q bands. But the basic dilation factor (usually called a_0) is fixed at 2. There exist ways to build wavelet bases for rational a_0's. But the a_0 needed to create the musical scale is $\sqrt[12]{2}$, an irrational number. This means that the standard tool for building wavelet bases (the multirresolution analysis) must be abandoned.

Regarding the restrictions of discrete dyadic wavelets, in [TOR/99], p.22, Torresani says: "The connection between continuous and discrete wavelet systems is not completely understood. ... The multirresolution approach seems to be also extremely constrained by algebraic arguments, which should be developed further." And in [DAU/92], p.16, Daubechies says: "Although the constructive method for orthonormal wavelet bases, called multirresolution analysis, can work only if a_0 is rational it is an open question whether there exist orthonormal wavelet bases (necessarily not associated with a multirresolution analysis), with good time-frequency localization, and with irrational a_0." It is also worth noting that in the cover of [STR/97] appears a piece of sheet music with several notes, as a metaphor of a dyadic wavelet base. But although each octave has 12 different notes, and 5 octaves are

shown, only one note per octave is included (the first one, called C). So, all the notes displayed have a frequency $2^i f_0$ for some f_0 with integer i. This is enough to suggest the need of generalizing wavelet bases so we can also represent the other notes of the scale.

This work presents a new family of bases, specially designed to fit the bandwidth of each tone of the musical scale.

The first task is to choose a tiling of the time-frequency plane, with constant Q bands, and with one band covering the frequencies of each tone of the scale, and where each band is critically sampled.

The following problem is to find a wavelet that is localized on a tile. Appropriate dilations and displacements will allow to position the wavelet over any tile of the partition. It is impossible t confine a wavelet strictly to a tile, because no signal can have compact support both in the time and in the frequency domains. In fact, to have a good decay in one domain, infinite support is needed in the other. Therefore the best wavelets for this problem have infinite support in both domains. It i however possible to choose a better localization in one domain, accepting a worse localization in the other. The ideal frequency response would be a step: 1 inside the tile and 0 outside it. This would carry a complete lack of temporal localization (as it happens in the Fourier Transform) and is no useful. Anyway, the bases presented in this work have very little frequency overlap between different bands, to allow good discrimination of the tones in the signal being processed. This also means tha there is a significant temporal overlap between the elements of the base that belong to the same band. The chosen tradeoff between tone discrimination and time resolution is expected to be adequate. The bases built are orthonormal, meaning that the transform and reconstruction of signals are easy to do.

The results obtained can be compared with those from [BER/99]. In this work, the authors address the more general problem of creating a base for any tiling of the time-frequency plane and building a base that can approximate it. The work presented here is of more limited scope, focusing only on music processing, but the frequency localization achieved is better. This means that the tone discrimination will be better and the results of processing will be closer to what the user expects.

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